Problem

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PROBLEM: Let k and n be positive integers, and let $m = \min\{k, n\}$. Prove that for each integer $y \in [0, m]$, we have

$$\sum_{x=y}^{m} \left\{ {n \atop x} \right\} (x)_y \, (k-y)_{x-y} = \sum_{i=0}^{y} \binom{y}{i} (-1)^i (k-i)^n.$$

Here, for $a \in \mathbb{R}$ and $b \in \mathbb{Z}^+$, we have $(a)_b := a(a-1)\cdots(a-b+1)$ and $(a)_0 := 1$, and $\left\{ {n \atop x} \right\}$ is the Stirling number of the second kind, i.e. the number of ways to partition the set $[n] := \{1, 2, \ldots, n\}$ into x nonempty blocks.