

1. A researcher studying butterflies in St. Joseph County makes observations at random places throughout the county. She has noted that at 60% of the locations she observes Meadow Fritillaries, and that at 15% of the locations West Virginia Whites are observed. However neither were observed at 33% of the locations.

Consider the following events:

$A$  : Meadow Fritillaries are observed

$B$  : West Virginia Whites are observed

What is the probability the researcher will observe both types of butterflies at a given location?

Are  $A$  and  $B$  mutually exclusive? Explain your reasoning.

Are  $A$  and  $B$  independent? Please explain how you reached your decision.

2. A casino has the following game. The player begins by rolling a fair 6-sided die. If it is a 6, the player wins \$10 and the game is over. Otherwise the player rolls again, and if the roll is a 6 then the player wins \$20 and the game is over. Otherwise the player rolls for a third time. If it is a 6, the player wins \$30. If not the game is over and the player wins nothing.

Let  $X$  be the amount the money the player wins. Write the probability distribution for  $X$ .

What is the expected value and variance of  $X$ ?

What is the smallest whole dollar amount the casino could charge to play, and still make a profit? (Remember, the casino wins only if the player loses)

3. (10 points) There are three fair six-sided dice. What is the probability of rolling them and having at least two dice match (that is, be equal to each other)?

4. A giant urn contains 12 red balls, 17 white balls, and a single green ball. You may leave your answer in terms of  $P(n, k)$ ,  $C(n, k)$  and  $n!$ .  
I draw three balls out of the urn, what is the probability that one of them is the green ball?

My brother came to visit, and he brought his giant urn. His contains 50 red balls and 25 white balls. We can't tell them apart, so we randomly choose an urn and pull two balls out of it.

What is the probability that both the balls are red?

Given that both the balls are red, what is the probability that we drew from my brother's urn?

5. There are about 8,300 undergraduates at Notre Dame, and about 1000 of them are from the western United States. Suppose we choose 20 undergraduates at random. Let  $X$  be the number of people from the west in our sample.

What is the expected value ( $\mu$ ) of  $X$ ?

What is the variance of  $X$ ?

What is the probability the group would have less than  $\mu$  people from the west?

6. (10 points) On my Hawaii sight-seeing cruise I am told that they see on average 2 dolphins per hour. The tour is 1.5 hours long. Let  $X$  be the number of dolphins spotted on the trip. Use a Poisson distribution to model  $X$ .

What is the value of parameter  $\mu$ ?

What is the probability I will see at least two dolphins?