

§1. Let X be normal with mean $\mu_X = 0$ and $\sigma_X = 2$. What is the probability X is less than 0.5?

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

§2. Suppose I roll a fair six-sided die 150 times. (whew!) Let X be the number of times I rolled a 6. Which is a good estimate of $P(X > 30)$? (Z is a standardized normal random variable.)

- (a) $P(Z \geq \frac{29.5 - 25}{20.83})$ (b) $P(Z \geq \frac{29.5 - 25}{4.56})$
(c) $P(Z \geq \frac{30.5 - 25}{4.56})$ (d) $P(Z \geq \frac{30 - 25}{20.83})$
(e) $P(Z \geq \frac{30 - 3}{2.48/\sqrt{150}})$

§3. Consider the function

$$f(x) = \begin{cases} \frac{2}{9}x, & \text{if } 0 \leq x \leq c \\ 0, & \text{otherwise} \end{cases}$$

(1) What value of c makes f a probability density function?

- (a) 2 (b) 1 (c) $\sqrt{2}$ (d) 3 (e) $9/2$

(2) Using the value of c you found in the previous question, let X be a random variable having f as its probability density function. What is the mean of X ?

- (a) 1 (b) 1.5 (c) 2 (d) 2.5 (e) 0.5

(3) What is the standard deviation of X ?

- (a) $1/\sqrt{2}$ (b) $3/4$ (c) 1 (d) $\sqrt{2}$ (e) \sqrt{c}

§4. Voter turnout is usually lower for mid-term elections. We wish to see if the percentage of young voters (18–29 years old) in South Bend is different from the population as a whole. We sample a randomly selected group of young people and a group from the population at large.

(1) Which test statistic is appropriate for this task?

(a) $(X - \mu)/\sigma$

(b) \bar{x}

(c) \hat{p}

(d) $\bar{x}_1 - \bar{x}_2$

(e) $\hat{p}_1 - \hat{p}_2$

(2) Suppose both samples are the same size n . How large should n be if we want to be within 3% of the true value 90% of the time?

(a) 30

(b) 752

(c) 1504

(d) 925

(e) not enough information

- (3) We did the survey, but we couldn't get as many samples as we wished. Which expression below gives a 90% confidence interval for the data below?¹

| | Young People | At Large |
|----------|--------------|----------|
| surveyed | 257 | 313 |
| voted | 83 | 94 |

- (a) $0.31 \pm (1.645)(0.019)$ (b) $0.31 \pm (1.96)(0.019)$
- (c) $0.023 \pm (1.645)(0.039)$ (d) $0.023 \pm (1.645)(0.019)$
- (e) $0.5 \pm (1.285)(83 + 94)/(\sqrt{257 + 313})$

¹This data is made up. I don't know how many young people voted in last Tuesday's election.

- §5. (Short Answer) A packaging machine fills boxes of nails by weight. Each box should be 5 pounds. We take a sample of 50 boxes and find the average weight per box is 4.89 pounds, with a standard deviation of 0.3 pounds. What is a 95% confidence interval for the average weight of boxes filled by the machine?

Using the 95% confidence interval can you conclude whether the machine is working correctly? Why or why not?

§6. (Short Answer) In Toronto I took a streetcar to work everyday. If the streetcar comes every 10 minutes, then the amount of time I need to wait is uniformly distributed between 0 and 10. Let T be the amount of time I spent waiting for the streetcar over 60 trips.

(1) What is the approximate distribution of T ? Justify your answer.

(2) What is the mean of T ?

(3) What is the standard deviation of T ?