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NOT EVERY CO-EXISTENTIAL MAP IS CONFLUENT

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ABSTRACT. A continuous surjection between compacta is *co-existential* if it is the second of two maps whose composition is a standard ultracopower projection. Co-existential maps are always weakly confluent, and are even monotone when the range space is locally connected; so it is a natural question to ask whether they are always confluent. Here we give a negative answer.

This is an interesting question, mainly because of the fact that most theorems about confluent maps have parallel versions for co-existential maps—notably, both kinds of maps preserve hereditary indecomposability. Where the known parallels break down is in the question of chainability. It is a celebrated open problem whether confluent maps preserve chainability, or even being a pseudo-arc; however, as has recently been shown [7], co-existential maps do indeed preserve both these properties.

1. INTRODUCTION

Co-existential maps are defined using topological ultracopowers in an exact mirroring of how one characterizes the existential embeddings of model theory in terms of ultrapowers. (See, e.g., [3] for a full explanation.) Briefly, if X is a *compactum* (i.e., a compact Hausdorff space) and $\mathcal{D}$ is an ultrafilter on a set I (viewed as a discrete topological space), then we let $p : X \times I \to X$ and $q : X \times I \to I$ be the standard projection maps. The $\mathcal{D}$-*ultracopower* of X is denoted $XI\backslash\mathcal{D}$, and is the inverse image of the point $\mathcal{D} \in \beta(I)$ with respect to the Stone-Čech lift

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$q^\beta : \beta(X \times I) \to \beta(I)$. The restriction of $p^\beta : \beta(X \times I) \to X$ to $XI\backslash\mathcal{D}$ is a continuous surjection, is denoted $p_{X,\mathcal{D}}$, and is called the *codiagonal* map associated with the given ultracopower.

$$\begin{array}{ccccc} XI\backslash\mathcal{D} & \xrightarrow{\subseteq} & \beta(X \times I) & \xrightarrow{q^\beta} & \beta(I) \\ & \searrow p_{X,\mathcal{D}} & \downarrow p^\beta & & \\ & & X & & \end{array}$$

Now we may define a function $f : X \to Y$ between compacta to be a *co-existential map* if there is an ultracopower $YI\backslash\mathcal{D}$ and a continuous surjection $g : YI\backslash\mathcal{D} \to X$ such that $f \circ g = p_{Y,\mathcal{D}}$.

$$\begin{array}{ccc} & & YI\backslash D \\ & g \swarrow & \downarrow p_{Y,\mathcal{D}} \\ X & \xrightarrow{f} & Y \end{array}$$

In particular any ultracopower codiagonal map is co-existential. In the sequel, a *continuum* is a compactum that is connected; a *subcontinuum* of a space is a subspace that is also a continuum. We take [10] as our main source for continuum theory, noting that we differ from that text only to the extent that our continua do not have to be metrizable. The following is a short list of examples of how co-existential maps arise.

Examples 1.1. (1) (See Remark 1.4 in [4]) *If $\mathcal{A}$ and $\mathcal{B}$ are lattice bases for compacta X and Y, respectively, and if there is an existential embedding from $\mathcal{B}$ into $\mathcal{A}$, then that embedding naturally induces a co-existential map from X onto Y.*

(2) (Theorem 2.7 in [2]) *A function from an arc to a compactum is a co-existential map if and only if the range is an arc and the map is a continuous monotone surjection.*

(3) (Corollaries 2.17 and 2.18 in [5]) *If G is a connected topological graph, then there is a co-existential map from G onto a simple closed curve (resp., an arc) if and only if G is not a topological tree (resp., there is a point of G whose complement has exactly two components).*

(4) (Proposition 6.2 in [2]) *If Y is a totally disconnected compactum with no isolated points, then every continuous map from a compactum onto Y is co-existential.*

(5) (Theorem 6.1 in [2] and Corollary 4.13 in [4]) *Every nondegenerate continuum X is a continuous image of a hereditarily indecomposable continuum*

Y, of the same weight as X and having (Lebesgue-Čech) covering dimension one, such that every continuous map from a continuum onto Y is co-existential.

A continuous surjection $f : X \to Y$ is termed *confluent* (resp., *weakly confluent*) if every (resp., some) component of $f^{-1}[K]$ is mapped onto K via f whenever K is a subcontinuum of Y. f is *monotone* if $f^{-1}[K]$ itself is connected; f is *open* if $f[U]$ is open in Y for each U that is open in X. The class of confluent mappings contains both those of open and of monotone maps; our main interest here is in showing that it does not contain the class of co-existential maps. The following summarizes the main known features of co-existential maps.

Theorem 1.2. (1) (Theorems 2.4 and 2.7 in [2]) *Co-existential maps are continuous surjections that are weakly confluent; in the case where the range is locally connected, they are monotone.*

(2) (Proposition 2.5 in [2], Proposition 4.2 in [4]) *Co-existential maps preserve the topological properties of: being infinite, being disconnected, being totally disconnected, being an indecomposable continuum, being a hereditarily indecomposable continuum, and being a hereditarily decomposable continuum (but not being a decomposable continuum).*

(3) (Example 5.4 (2) in [7]) *Co-existential maps preserve the property of being a chainable continuum.*

(4) (Example 5.2 (3) and Remark 5.5 (3) in [7]) *A co-existential image of a pseudo-arc is a pseudo-arc.*

(5) (Theorem 2.6 in [2]) *Co-existential maps do not raise covering dimension.*

(6) (Corollary 5.4 in [4]) *Co-existential maps do not raise multicoherence degree in continua.*

(7) (Theorem 5.3 in [6]) *Co-existential maps preserve the following properties of continua: unicoherence, hereditary unicoherence, weak hereditary unicoherence, and strong unicoherence.*

2. ANATOMY OF A CODIAGONAL MAP

Let $F(X)$ be the bounded lattice of all closed subsets of a compactum X. A sublattice of $F(X)$ that is also a closed-set base for X is called a *lattice base* for X. The set of complements of members of a lattice base is itself a lattice, as well as an open-set base, and is referred to as an *open lattice base* for X.

If $\mathcal{D}$ is an ultrafilter on a set I, we may view (see [3]) the points of $XI\backslash\mathcal{D}$ as consisting of maximal filters in the lattice $F(X)^I/\mathcal{D}$, the ultrapower of the lattice $F(X)$ via $\mathcal{D}$. By way of notation and explanation: if $\langle S_i : i \in I\rangle$ is an

I-indexed family of sets, let $\prod_{\mathcal{D}} S_i$ be the set-theoretic ultraproduct of the family (see, e.g., [8]). Points of the ultraproduct are $\mathcal{D}$-equivalence classes of members of the cartesian product $\prod_{i\in I} S_i$, where two I-tuples s and t are $\mathcal{D}$-equivalent if $\{i \in I : s_i = t_i\} \in \mathcal{D}$. When each S_i is endowed with some algebraic operations, there is a natural way to induce these operations on the ultraproduct. And when the original operations are those of a lattice, so too are the induced operations. In the case each S_i is equal to a single set S, the ultraproduct $\prod_{\mathcal{D}} S_i$ is referred to as an *ultrapower*, and is denoted $S^I/\mathcal{D}$. So, in particular, when $S = F(X)$ is a closed-set lattice, there is a naturally-defined lattice structure on $S^I/\mathcal{D}$, and it makes sense to define $XI\backslash\mathcal{D}$ as above.

Let us now regard each S_i as a subset of a compactum X, and denote by $(\prod_{\mathcal{D}} S_i)^\sharp$ the collection $\{\mu \in XI\backslash\mathcal{D} : \text{some member of } \mu \text{ is contained in } \prod_{\mathcal{D}} S_i\}$. Then the standard lattice base for the ultracopower comprises the sets $(\prod_{\mathcal{D}} C_i)^\sharp$, where the sets C_i, $i \in I$, are all closed in X. Moreover, if $U_i \subseteq X$ is open, $i \in I$, then it is easy to show that $(\prod_{\mathcal{D}} U_i)^\sharp$ is open in $XI\backslash\mathcal{D}$. (Indeed, it is the complement, in $XI\backslash\mathcal{D}$, of $(\prod_{\mathcal{D}}(X \setminus U_i))^\sharp$.) Finally, for any $\mu \in XI\backslash\mathcal{D}$ and $x \in X$, $x = p_{X,\mathcal{D}}(\mu)$ if and only if, for each open neighborhood U of x in X, we have $\mu \in (U^I/\mathcal{D})^\sharp$.

By way of simplifying notation in the sequel: when there is only one ultrafilter under discussion in a particular argument, we let $S^\sharp$ abbreviate $(S^I/\mathcal{D})^\sharp$.

Lemma 2.1. *Let $\mathcal{B}$ be an open lattice base for a compactum X, with $\mathcal{D}$ an ultrafilter on a set I. Let $p = p_{X,\mathcal{D}}$.*

(1) *If $C \subseteq X$ is closed, then $C^\sharp \subseteq p^{-1}[C]$, and $p[C^\sharp] = C$.*
(2) *If $U \subseteq X$ is open, then $p^{-1}[U] = \bigcup\{\overline{B}^\sharp : B \in \mathcal{B} \text{ and } \overline{B} \subseteq U\} \subseteq U^\sharp$, and $p[U^\sharp] \subseteq \overline{U}$.*
(3) *If $C \subseteq X$ is closed, then $p^{-1}[C] = \bigcap\{B^\sharp : B \in \mathcal{B} \text{ and } C \subseteq B\} = \bigcap\{\overline{B}^\sharp : B \in \mathcal{B} \text{ and } C \subseteq B\}$.*
(4) *If $U \subseteq X$ is open, $C \subseteq X$ is closed, and $U \subseteq C$, then $p^{-1}[U] \subseteq C^\sharp$.*

Proof. Ad (1): Assume $\mu \notin p^{-1}[C]$. Then there is some open neighborhood U of $p(\mu)$ with $\overline{U} \cap C = \emptyset$. Hence $(\overline{U}^I/\mathcal{D}) \cap (C^I/\mathcal{D}) = \emptyset$; and since $\overline{U}^I/\mathcal{D} \in \mu$, it follows that $\mu \notin C^\sharp$.

Clearly from the first paragraph, we know $p[C^\sharp] \subseteq C$. If $x \in X$, then $x = p(\mu)$ for any $\mu \in X^I\backslash\mathcal{D}$ such that $\{x\}^I/\mathcal{D} \in \mu$. If $x \in C$, then any such μ must lie in $C^\sharp$.

Ad (2): Suppose that $B \in \mathcal{B}$, $\overline{B} \subseteq U$, and $\mu \in \overline{B}^\sharp$. Then $p(\mu) \in \overline{B}$, by (1) above; so $p(\mu) \in U$. Thus $\mu \in p^{-1}[U]$. Now suppose $\mu \in p^{-1}[U]$. Then there is some $B \in \mathcal{B}$, with $p(\mu) \in B \subseteq \overline{B} \subseteq U$. Thus $\mu \in B^\sharp \subseteq \overline{B}^\sharp$. The union on the right-hand side is obviously contained in $U^\sharp$. Finally, $p[U^\sharp] \subseteq p[\overline{U}^\sharp] = \overline{U}$, by (1).

Ad (3): To start with, the two big intersections are equal, on account of the regularity of compacta. Suppose $\mu \in p^{-1}[C]$, with $B \in \mathcal{B}$, and $C \subseteq B$. Then $\mu \in B^\sharp$, by (2). If $\mu \notin p^{-1}[C]$, then we can find $B \in \mathcal{B}$ with $C \subseteq B$ and $p(\mu) \notin \overline{B}$. Then $\mu \notin p^{-1}[\overline{B}] \supseteq \overline{B}^\sharp$, by (1).

Ad (4): We have the inclusion $U^\sharp \subseteq C^\sharp$; apply (2) above.

□

Remarks 2.2. (1) From well-known properties of ultracopowers (see [3]), if C is a subcompactum of X, then $CI\backslash\mathcal{D}$ is homeomorphic to the subcompactum $C^\sharp$ of $XI\backslash D$. Ultracopowers of continua are themselves continua; hence, by Lemma 2.1(1), if C is a subcontinuum of X, then $C^\sharp$ is a subcontinuum of $XI\backslash\mathcal{D}$ that witnesses the weak confluence of the codiagonal map.

(2) If X is locally connected, then X has an open lattice base $\mathcal{B}$ that consists if finite unions of connected sets. From this fact, and Lemma 2.1(3), we may infer that any ultracopower codiagonal map onto X must be monotone.

3. codiagonal maps that are not open

In this section we show that ultracopower codiagonal maps are very rarely open maps.

Proposition 3.1. *Let X be a metrizable compactum, with $\mathcal{D}$ a countably incomplete ultrafilter on a set I. If $U \subseteq X$ is open, then $p_{X,\mathcal{D}}[U^\sharp] = \overline{U}$.*

Proof. As above, we let $p = p_{X,\mathcal{D}}$. By Lemma 2.1(2), we have only to show that the right-hand side is contained in the left. So suppose that $x \in \overline{U}$, and that $\langle x_n : n \in \omega\rangle$ is a sequence of points in U, converging to x. Since $\mathcal{D}$ is countably incomplete, we have a nested sequence $I = J_0 \supseteq J_1 \supseteq \dots$ of sets such that $J_n \in \mathcal{D}$, for all $n \in \omega$, and $\bigcap_{n\in\omega} J_n = \emptyset$. Then for each $i \in I$, we may define $\alpha(i)$ to be the largest $n \in \omega$ such that $i \in J_n$. For each $i \in I$, define x_i to be

$x_{\alpha(i)}$. Let $\mu \in XI\backslash\mathcal{D}$ be such that $\prod_{\mathcal{D}}\{x_i\} \in \mu$. It follows that if V is any open neighborhood of x, then $\{i \in I : x_i \in V\} \supseteq J_{\alpha(i)} \in \mathcal{D}$. Hence $\prod_{\mathcal{D}}\{x_i\} \subseteq V^I/\mathcal{D}$; i.e., $\mu \in V^\sharp$. Thus $p(\mu) = x$. Since $\prod_{\mathcal{D}}\{x_i\} \subseteq U^I/\mathcal{D}$, we have $\mu \in U^\sharp$. □

Theorem 3.2. *Let X be an infinite metrizable compactum, with $\mathcal{D}$ a countably incomplete ultrafilter on a set I. Then $p_{X,\mathcal{D}}$ is not an open map.*

Proof. Since X is an infinite metrizable compactum, it has convergent sequences that are not eventually constant. It is well known [12] that X is therefore not *extremally disconnected*; i.e., there is an open $U \subseteq X$ whose closure is not open. By Proposition 3.1, $p_{X,\mathcal{D}}$ takes the open set $U^\sharp$ onto $\overline{U}$. Hence this codiagonal map is not open. □

4. Codiagonal maps that are not monotone

In this section we show how to produce ultracopower codiagonal maps (onto continua that may be taken to be metrizable) that are confluent, but neither open nor monotone. We first define a *co-existentially closed* (resp., *confluently closed*) *continuum* to be a continuum that is only a co-existential (resp., confluent) image of other continua. Recall that a continuum (or compactum) is *hereditarily indecomposable* if no two of its subcontinua overlap without one being contained in the other. The pseudo-arc is the best-known example of a hereditarily indecomposable continuum; the following is a somewhat expanded version of what we stated in Example 1.1(5).

Theorem 4.1.
(1) (Theorem 6.1 in [2]) *Every continuum is a continuous image of a co-existentially closed continuum, of the same weight.*
(2) (Corollary 4.13 in [4]) *Every co-existentially closed continuum is hereditarily indecomposable, and of covering dimension one.*
(3) (see [10]) *A continuum is confluently closed if and only if it is hereditarily indecomposable.*

Remark 4.2. Compare our definition of confluently closed continuum with that of the better-known Class(C), studied in A. Lelek's Houston Topology Seminar of the 1970s, where the context is *metrizable* continua. However, since the metrizability assumption may be eliminated from the characterization Theorem 4.1 (3) above, it turns out that Class(C) comprises the confluently closed continua that are metrizable. The term *confluently closed* is chosen here to emphasize the obvious analogy with the class of co-existentially closed continua; which, in turn, is so named because of the obvious dual analogy with the notion of *existentially* *closed model of a first-order theory* (see, e.g., [8]). In view of Theorem 4.1 (2,3), we see that every co-existentially closed continuum is also a confluently closed continuum. This adds further interest to the question of whether co-existential maps are always confluent.

Before stating the next theorem, we remind the reader of a basic topological construction. Let X and Y be spaces, with points $a \in X$ and $b \in Y$ given. Then the *wedge sum* $(X, a) \vee (Y, b)$ is the quotient space $[(X \times \{0\}) \cup (Y \times \{1\})]/\sim$, where $\sim$ is the equivalence relation whose only nontrivial equivalence class is $\{(a, 0), (b, 1)\}$. This is the classic way of "spot welding" two continua together to obtain a new one.

In light of Theorem 4.1(1), the hypothesis of the next theorem is nonvacuous.

Theorem 4.3. *Let X be a metrizable co-existentially closed continuum. Then there is an ultrafilter $\mathcal{D}$ on a set I such that $p_{X,\mathcal{D}}$ is confluent, but neither open nor monotone.*

PROOF. We first note that (Theorem 4.1(2)) since X is a co-existentially closed continuum, it must be nondegenerate, hence infinite. Pick a point $a \in X$, and define Y to be the wedge sum $(X, a) \vee (X, a)$. Let $f : Y \to X$ take the equivalence class containing (x, i) to x. Then Y is a continuum and f is not monotone. But, because X is a co-existentially closed continuum, f is a co-existential map. Thus there is an ultrafilter $\mathcal{D}$ on a set I, and a continuous surjection $g : XI\backslash\mathcal{D} \to Y$, such that $f \circ g = p = p_{X,\mathcal{D}}$. Since f is not monotone, it follows immediately that p is not monotone either. By Theorem 4.1(2,3), p is a confluent map; it remains to show p is not open.

By Theorem 3.2, p will be shown not to be open once we establish that $\mathcal{D}$ is a countably incomplete ultrafilter. First, we know $\mathcal{D}$ is nonprincipal. Otherwise p would be a homeomorphism; but we already know p is not monotone. So if it were the case that $\mathcal{D}$ is countably complete, then, by well-known set-theoretic facts (see, e.g., [8]), the only way for p not to be a homeomorphism would be for the cardinality of X to exceed a measurable cardinal. But X is metrizable, it has the cardinality of the continuum, and this scenario is impossible. Thus $\mathcal{D}$ must be countably incomplete after all. $\square$

5. CODIAGONAL MAPS THAT ARE NOT CONFLUENT

In this section we provide a general method for producing ultracopower codiagonal maps that are not confluent.

Theorem 5.1. *Let X be a co-existentially closed continuum, with Y any nondegenerate continuum. Then there is an ultrafilter $\mathcal{D}$ on a set I and a wedge sum Z of X and Y such that $p_{Z,\mathcal{D}}$ is not confluent.*

PROOF. Given the co-existentially closed continuum X, we use the construction in the proof of Theorem 4.3 to find an ultrafilter $\mathcal{D}$ on a set I such that $p_{X,\mathcal{D}}$ is not monotone. Fix $a \in X$ such that $p_{X,\mathcal{D}}^{-1}[\{a\}]$ is disconnected, and let $b \in Y$ be arbitrary. We then set $Z = (X,a) \vee (Y,b)$. Without loss of generality, we may assume X and Y are disjoint, and we give $X \cup Y$ the obvious disjoint union topology. Let $q : X \cup Y \to Z$ be the standard quotient map. Then we set $A = q[X]$ and $B = q[Y]$, so that $q|X$ (resp., $q|Y$) is a homeomorphism between X and A (resp., Y and B). Let $c = q(a) = q(b)$, so that $A \cap B = \{c\}$. Because we are working with only one ultrafilter in this argument, we use $(\ )^\sharp$ to abbreviate both $((\ )^I/\mathcal{D})^\sharp$ and $(\ )I\backslash\mathcal{D}$. But since there are several codiagonal maps present, we drop the ultrafilter subscript only.

Our claim, then, is that B witnesses the failure of the confluence of p_Z. First of all, $U = Z \setminus A$ is a nonempty open subset of Z, contained in B. Hence, by Lemma 2.1(4), $p_Z^{-1}[U] \subseteq B^\sharp$. Now $B^\sharp$, being a subcontinuum of $Z^\sharp$, is contained in a component of $p_Z^{-1}[B]$, by Lemma 2.1(1). Consequently no other component of $p_Z^{-1}[B]$ can map onto B. It therefore remains to show that $p_Z^{-1}[B]$ is disconnected.

Let $q^\sharp : (X \cup Y)^\sharp \to Z^\sharp$ be the *ultracopower lift* of q; i.e., the map defined by the condition $\prod_{\mathcal{D}} C_i \in q^\sharp(\mu)$ if and only if $\prod_{\mathcal{D}} q^{-1}[C_i] \in \mu$. Then we have a commutative square of maps, defined by the equality $q \circ p_{X\cup Y} = p_Z \circ q^\sharp$. We may regard $(X \cup Y)^\sharp$ as naturally homeomorphic to $X^\sharp \cup Y^\sharp$, so we may abuse notation slightly and write the commutativity equation as $q \circ (p_X \cup p_Y) = p_Z \circ q^\sharp$. It is easy to show that $q^\sharp|X^\sharp$ (resp., $q^\sharp|Y^\sharp$) is a homeomorphism between $X^\sharp$ and $A^\sharp$ (resp., $Y^\sharp$ and $B^\sharp$). Also we have $A^\sharp \cap B^\sharp = (A \cap B)^\sharp = \{c\}^\sharp$, a singleton set. So indeed $Z^\sharp$ is naturally homeomorphic to the wedge sum $(X^\sharp, \mu_a) \vee (Y^\sharp, \mu_b)$, where μ_a and μ_b are the single elements of $\{a\}^\sharp$ and $\{b\}^\sharp$, respectively. Thus $p_Z^{-1}[B]$ is naturally homeomorphic to the wedge sum $(p_X^{-1}[\{a\}], \mu_a) \vee (Y^\sharp, \mu_b)$, a disconnected set because $p_X^{-1}[\{a\}]$ is. □

Recall that a map is called *light* if the inverse image of every point in the range is totally disconnected. Because of Theorem 5.1, the hypothesis of the following result is nonvacuous.

Proposition 5.2. *Suppose X is any compactum that is the image of a compactum under a co-existential map that is not confluent. Then X is also the image of a compactum under a co-existential light map that is not confluent.*

PROOF. Let $f : Y \to X$ be a co-existential map that is not confluent. Then (see, e.g., [10, 11]) there is a continuum Z, a monotone surjection $g : Y \to Z$, and a light surjection $h : Z \to X$ such that $f = h \circ g$. Then h is easily seen (Corollary 2.7 in [1]) to be a co-existential map. Since f is not confluent and confluence is closed under mapping composition, we infer that h is not confluent either. □

Added in Proof: Using model-theoretic methods, particularly the Löwenheim-Skolem theorem (see [3] for details), one may start with a nonconfluent (nonmonotone, nonopen) codiagonal map $p_{X,\mathcal{D}}$ and derive a map $f : Y \to X$, where: (i) f is nonconfluent (nonmonotone, nonopen); (ii) Y has the same weight as X; and (iii) f is co-existential. (Indeed, f is *co-elementary*; i.e., there is a homeomorphism between ultracopowers $h : YJ\backslash\mathcal{E} \to XI\backslash\mathcal{D}$, such that $p_{X,\mathcal{D}} \circ h = f \circ p_{Y,\mathcal{E}}$.) Also, in a recent paper [9], K. P. Hart has produced a simple example of a nonconfluent co-existential map between two unidimensional planar continua.

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